CONTENTS

	Page	
SUMMARY	1	1/A5
INTRODUCTION	1	1/A5
THEORY OF PROPAGATION OF SOUND OVER GROUND	3	1/A7
EXPERIMENTS ON PROPAGATION OF SOUND OVER GROUND	5	1/A9
GROUND IMPEDANCE	8	1/A12
Methods of Measuring Impedance	8	1/A12
Impedance tube	8	1/A12
Free field	8	1/A12
Inclined track	9	1/A13
Direct pressure-velocity measurements	ģ	1/A13
Pressure-gradient measurements	10	1/A14
Calculation from flight noise data	10	1/A14
	10	1/A14
Ground-Impedance Data	10	1/415
STANDARD PRACTICE IN CORRECTING FOR GROUND EFFECTS	12	1/B2
Modification of Physical Environment	12	1/B2
Flush-mounted microphones	12	1/B2
Raised microphones	13	1/B3
Gravel and similar partly controlled impedance surfaces	13	1/B3
Numerical Ground-Effects Correction	13	1/B3
Data correction with known ground impedance	13	1/B3
	14	1/B4
Miscellaneous techniques		
Analytic correction methods	14	1/B4
Cepstral technique	15	1/85
RECOMMENDED PREDICTION METHOD	15	1/B5
PROBLEMS IN APPLICATION	21	1/811
	_	1/811
Finite Source	21	1/811
Filter Bandwidth	21	
Partial Coherence	22	1/B12
Wind Gradients	24	1/B14
CONCLUDING REMARKS	24	1/B14
APPENDIX - RELATIVE IMPORTANCE OF PLANE WAVE APPROXIMATION	26	1/C2
REFERENCES	28	1/04
PTCHPPS	32	1/C8

8461 8 933 NASA Technical Paper 1104



Prediction of Ground Effects on Aircraft Noise

S. Paul Pao, Alan R. Wenzel, and Paul B. Oncley

JANUARY 1978





NASA Technical Paper 1104

Prediction of Ground Effects on Aircraft Noise

S. Paul Pao
Langley Research Center
Hampton, Virginia
Alan R. Wenzel
Institute for Computer Applications
in Science and Engineering
Langley Research Center
Hampton, Virginia

Paul B. Oncley

MAN - Acoustics and Noise, Inc.
Seattle, Washington



Scientific and Technical Information Office

SUMMARY

A unified method is recommended for predicting ground effects on aircraft noise. This method may be used in flyover noise predictions and in correcting static test-stand data to free-field conditions. The recommendation is based on a review of recent progress in the theory of ground effects and of the experimental evidence which supports this theory. This review shows that a surface wave, a recently discovered effect, must be included sometimes in the prediction method. Prediction equations are collected conveniently in a single section of the paper.

Methods of measuring ground impedance and the resulting ground-impedance data are also reviewed because the recommended method is based on a locally reactive impedance boundary model. Available data support a simple model of ground impedance, but these data have significant scatter and it is found that there is need for further data, classified by terrain types such as grassland and desert. Experiments wherein ground impedance and the ground effects on noise propagation are measured simultaneously are also needed.

Current practice of estimating ground effects are reviewed and consideration is given to practical problems in applying the recommended method. These problems include finite frequency-band filters, finite source dimension, wind and temperature gradients, and signal incoherence.

INTRODUCTION

The effects of propagation on noise radiated from aircraft were reviewed by Putnam (ref. 1) so as to develop a standard prediction method for use by the National Aeronautics and Space Administration. Putnam's review was one of several, referred to as "Key Technology Documents," which addressed different aspects of aircraft noise prediction. In the part of his review concerned with the ground effects on aircraft noise, Putnam gave concise descriptions of the ray acoustic analysis of sound reflections by surfaces, of sound attenuation over ground according to the Rudnick theory, and of the practical problems which are encountered in outdoor acoustic measurements. Putnam recommended empirical methods to predict ground effects on sound propagated from one ground station to another and from an aircraft to the ground.

The present paper updates the NASA ground-effects prediction method developed by Putnam and is designed to (1) improve the prediction methods for ground effects given in reference 1, (2) include methods for correcting aircraft engine test-stand noise measurements to free-field conditions, (3) present a unified analytical prediction method for both short-distance and long-distance propagation over ground, and (4) define the range of elevation angle within which ground effects may have a significant effect on the measurement of noise from aircraft in flight. The prediction scheme recommended in this paper is intended to supplant previous empirical techniques with a unified approach.

Significant advances have been made in recent years in understanding the effects of the ground surface on outdoor sound propagation. In particular, the rapid progress in the theory, which has occurred since 1974, has greatly enhanced the ability to make predictions. These advances in theory and experiment are summarized herein and form a basis for the recommended prediction procedures. Two recent review articles by Embleton et al. (ref. 2) and Piercy et al. (ref. 3) may be referred to for subjects related to ground effects which are beyond the scope of this paper.

A number of situations exist in which ground effects are important in aircraft noise prediction and measurement. These include the prediction of community noise resulting from aircraft operations, the correction of acoustic data in ground-based outdoor jet-engine noise measurement to free-field conditions, and the interpretation of data for aircraft sideline and flyover noise. Empirical methods have been devised to deal with each of these situations.

The ground surface is usually modeled as an infinite flat plane on which a normal impedance boundary condition is prescribed. This model appears to be adequate for most situations of practical interest. Real media such as sand, soil (with or without grass cover), and snow are porous with high internal flow resistance and have poor wave propagation characteristics. For such media, the assumption of a locally reacting surface is reasonable. The strongest support for the validity of the impedance boundary theory comes from the cumulative experimental observations of ground effects in the past three decades. Indeed, Piercy et al. (ref. 3) have indicated that there appears to be little need for a more elaborate model for the description of ground effects. Therefore, most of the subsequent discussion herein is based on this theoretical model.

In some situations the impedance model is inadequate. An example is ground covered with a thick, dense layer of vegetation (ref. 4). In this case, the vegetation layer should be regarded as an acoustic medium having a propagation constant somewhat different from that of air. A strong thermal gradient in the air above the ground can also create an apparent condition of sound propagation through a layered medium (ref. 5). A layered structure can also occur in the solid material of the ground. In the extreme case of clay or hard-packed soil, the ground should be regarded as an elastic medium. A detailed discussion of sound propagation in layered media can be found in Brekhovskikh (ref. 6), and many other discussions exist in the literature for these special conditions.

Perhaps the most important qualitative feature of the wave field, which has been brought out in the recent theoretical studies, is the surface wave. In contrast to the more familiar wave propagation phenomena in three-dimensional space (which is characterized by a more or less uniform radiation of energy in all directions, according to spherical spreading), the surface wave is effectively confined to a region near the boundary with a resultant decrease in amplitude with distance determined essentially by cylindrical spreading. An additional attenuation of the surface wave results from dissipation of energy at the boundary; however, this additional attenuation depends on the value of the normal impedance of the surface. If this additional attenuation is so small that the decay of the surface-wave amplitude is determined primarily by the cylindrical spreading, the surface wave becomes the dominant

mode of propagation. Such cases are usually associated with low-frequency waves for real ground surfaces.

The theories of propagation of sound over ground which have appeared since 1974 have now effectively superseded the earlier work of Ingard, Lawhead and Rudnick, and others. It is desirable to suggest at this point a new prediction method based on the recent theoretical work. In this paper, the theories are reviewed first. Then a review of the existing experimental evidence of ground effects is given. The techniques of ground-impedance measurement and existing ground-impedance data are reviewed also.

Many currently accepted practices for ground-effects corrections are empirical. Although the present paper recommends a prediction method based on theoretical analysis, some practical limitations typical of outdoor experiments remain. Therefore, a brief review is given of the general status of standard practices in the industry. This review serves as background information for the application of the recommended theoretical prediction method.

Since the present paper concentrates on the description of pure ground effects on sound propagation, the atmosphere above the ground is assumed to be homogeneous and motionless. Perturbations, such as wind, temperature gradients, and turbulence, have not been included in the discussions mentioned previously. However, variation from the ideal analytical model is certain to occur in practical applications. Therefore, some of the common problems in application are discussed.

THEORY OF PROPAGATION OF SOUND OVER GROUND

As stated previously, the theory of sound propagation over ground is regarded herein as equivalent to the theory of scalar wave propagation in a uniform half-space, which is bounded by an infinite plane on which a constant-impedance boundary condition is prescribed. (The geometry of this mathematical model is given in fig. 1. The coordinate system, source and receiver positions, the direct and the reflected paths of sound propagation, and the angle of incidence are defined in this figure.) Only the case of radiation by a time-harmonic point source is considered here. Nevertheless, even within this rather restricted context, this review is not intended to be exhaustive. Instead, it is limited primarily to those contributions to the theory which the authors feel are most significant in terms of fundamental understanding of the phenomenon or else are particularly useful for purposes of calculation.

Although published theoretical work on the problem of reflection of waves by an impedance boundary dates back at least to the 1944 paper by Morse and Bolt (ref. 7), the first analytical solutions of this problem to be given in a form suitable for numerical calculations were those of Ingard (ref. 8) and Lawhead and Rudnick (ref. 9). For more than two decades the papers by Ingard and Lawhead and Rudnick were regarded as the standard reference works in this field. Recently, however, a number of important contributions to the theory have appeared which largely supersede the earlier work of Ingard and Lawhead and Rudnick.

Of the recent contributions, the most noteworthy are those of Wenzel (ref. 10), Chien and Soroka (ref. 11), and Thomasson (ref. 12). Wenzel obtained asymptotic results, valid when both the source and receiver are near the boundary, for several limiting cases determined by the values of the wave number, propagation distance, and boundary admittance. He also found that, under certain conditions, a surface wave may be a significant component of the total wave field. Wenzel was apparently the first to point out the existence and implications of the surface wave in the general case, although Brekhovskikh (ref. 13) had noted previously that a surface wave occurs in the special case in which the boundary impedance is purely imaginary. Wenzel also noted that, in a certain asymptotic limit, his solution differs from Ingard's by just the surface-wave term. However, because these two authors used completely different methods, the reason for this discrepancy was not apparent.

Chien and Soroka also obtained, as did Wenzel, asymptotic solutions for various limiting cases. Their results, however, are valid more generally than are Wenzel's, particularly with regard to the case in which the source and receiver are not necessarily located near the boundary. Indeed, it appears that the various approximate solutions obtained by Chien and Soroka cover virtually all practical situations of interest, provided only that the field point is at least several wavelengths from the image source (a condition which is almost always met in practice). For this reason, their results are used herein as the basis for the recommended prediction method.

Chien and Soroka also found that, in a certain limiting case, their solution differs from Ingard's by a surface-wave term, thus confirming the discrepancy noted previously by Wenzel. However, the explanation offered by Chien and Soroka for this discrepancy, which they ascribed to the limited rape of validity of a certain asymptotic expansion used by Ingard in the evaluation of his approximate solution (see ref. 11, sec. 5.2.2), reveals that these authors did not fully grasp the significance of the missing surface-wave term in Ingard's solution, which is now known to be the result of a fundamental error in Ingard's analysis. This error was brought out in a subsequent paper by Thomasson (ref. 12), who showed that Ingard, in the process of deforming the path of integration of a certain contour integral occurring in his analysis, failed to take proper account of a pole in the integrand. It is this pole which gives rise to the surface-wave term.

The effect of Ingard's error is to restrict the range of validity of his solution to essentially those values of the boundary admittance for which the surface-wave term does not appear. A careful examination of Rudnick's solution shows that, although it was derived by a different approach than was Ingard's, it is subject to a similar restriction. In contrast, the more recent results, since they take explicit account of the surface-wave term, are not subject to this restriction and, indeed, are valid for all values of the boundary admittance.

The situation in acoustics, as regards the belated recognition of the importance of the surface wave, thus contrasts markedly with that in electromagnetic theory, in which context the surface wave has been the subject of much discussion, as well as considerable controversy, for over half a century. (See, e.g., refs. 14 and 15.)

In the same paper (ref. 12), Thomasson also gave an exact solution (essentially a corrected version of Ingard's exact solution) in the form of an integral which appears to be suitable for numerical calculation. This result is suggested as a supplement to the approximate results of Chien and Soroka in the recommended prediction method.

Thomasson (ref. 16) has also given an approximate solution, obtained from his exact solution, which is essentially a corrected version of Ingard's approximate solution. Recent contributions to the theory have also been given by Donato (refs. 17 and 18), Briquet and Filippi (ref. 19), and Van Moorhem (ref. 20).

EXPERIMENTS ON PROPAGATION OF SOUND OVER GROUND

Experiments regarding ground attenuation commenced as soon as the difference between the nature of wave systems for sound propagation over a flat boundary and those predicted by ray acoustics or plane wave theory was recognized. Rudnick and Oncley made outdoor measurements of ground attenuation effects at Duke University as early as 1945. The first significant experimental results on sound propagation over a finite-impedance boundary were those of Rudnick (ref. 21). He used commercial acoustic absorbing materials such as fiber glass in place of the ground as boundary surfaces. The experiments were later continued and extended by Lawhead and Rudnick (ref. 9). In both references, detailed analysis accompanied the experimental observations. Among the more important conclusions of these studies were the following:

- (1) The sound pressure amplitude is observed to attenuate according to r^{-2} at large values of kr away from the sound source where r is the radial distance and k is the wave number. This observation is distinctly different from the expected free-field attenuation of sound pressure amplitude with distance at the rate of r^{-1} , according to spherical spreading.
- (2) The distance at which the rate of attenuation r^{-2} is attained depends on frequency as well as the impedance of the ground. It takes place at a shorter distance for a surface with low impedance than for a surface with high impedance.
- (3) At a given horizontal distance away from the sound source, the minimum received sound pressure often occurs at a finite distance above the boundary.

Later studies by Ingard (ref. 22), Oleson and Ingard (ref. 23), and Wiener and Keast (ref. 24) confirmed much of these same basic phenomena of ground effects on sound propagation. In particular, Ingard made an important observation with respect to the geometrical configuration where both the source and the receiver are above the ground. He pointed out that the excess attenuation resulting from destructive interference between the direct and reflected sound paths was determined mainly by the impedance of the ground surface. The phase shift of ground reflection at near-grazing incidence can easily exceed 160° if computed according to typical values of acoustic impedance of natural ground surfaces. Maximum destructive interference can be reached by an additional geometrical path difference of 0.05 wavelength such that the total mismatch

between the direct and the reflected sound waves is half a wavelength at the point of measurement. The ground effect in such a geometry is heavily biased toward destructive interference. Consequently, the peak region of the excess attenuation spectrum is much broader and occurs at other frequencies than those predicted by using ray acoustics and a boundary of infinite impedance. In references 22 and 24 the observed peak of excess attenuation lies between 300 and 600 Hz.

As is typical of many of these field experiments, the ground effect is often observed together with prevalent atmospheric effects on sound propagation. The atmospheric nonuniformities usually include turbulence and often wind and temperature gradients. The experiments by Ingard have indicated that the ground attenuation effect can be studied and observed separately from most of the atmospheric effects.

Some important reference data on open-field ground effects were obtained by Parkin and Scholes (refs. 25 and 26). Measurements were made to study the combined effects of the ground and wind vector on sound propagation in a horizontal direction. There were actually two groups of data. Each of these was taken in a different airfield where an unobstructed open field was available. In both cases, a jet engine of 334-kN static thrust was used as the sound source. Acoustic measurements were made simultaneously at eight microphones positioned between 20 and 1100 m away from the sound source. Approximately 60 sets of measurements were taken in each group throughout the year.

The data obtained by Parkin and Scholes are important for the following reasons:

- (1) Attenuation of sound owing to ground effect is measured for a wide range of horizontal distances and provides a basis for the verification of theoretical calculations. In particular, the data sets with zero-vector wind conditions are representations of the ground effects, disturbed only by turbulence but not by other meteorological effects.
- (2) Typically, data scattering occurs in field measurements. Since measurements are repeated many times over the same ground environment, the statistical confidence level of the data set as a whole is greatly enhanced.
- (3) In cases where upwind conditions are prevalent, ray acoustic theories predict the formation of shadow zones. However, discussions in references 24 and 25 indicate that the observed boundary or shadow zone is frequency dependent. At low frequencies, such shadow-zone boundaries may not be observable. Recent theories indicate that such data may provide additional clues to the behavior of the surface-wave term (ref. 27).
- (4) Measurements were made at two different sites with similar arrangements and under similar weather conditions. Values of excess ground attenuation are different, whereas the overall characteristics remain similar. Furthermore, values of excess ground attenuation show noticeable change with season at each site. Hence, this data set can serve as an indirect reference for the characterization of ground-impedance values of grassland.

The determination of the acoustic impedance of typical ground surfaces is an important factor of the ground-effects prediction procedure. The studies of Delany and Bazley (refs. 28 and 29) marked the beginning of serious considerations of applying the theory to practical predictions. Based on their assumption that the acoustic impedance of soil and grassland can be modeled as typical porous material, calculations were made to predict values of excess ground attenuation. The results compared favorably with the data obtained by Farkin and Scholes. In most comparisons the geometrical configuration dealt with relatively short distances with the source and receivers above the ground. The surface wave was not considered since the calculations were based on the earlier theories developed by Ingard and Rudnick.

In references 30 and 31 a comprehensive approach was taken in the experimental observation of ground effects. Experiments were designed to observe both short-range (less than 20 m) and long-range (over 300 m) ground effects on sound propagation. The ground impedance was measured together with the acoustic data. Hence, theoretical predictions can be quantitatively correlated to the acoustic measurements. In the short-range experiments, good agreement is obtained between theory and experiments, regarding the amplitude and location of the interference patterns. However, local discrepancies at a specific frequency can be several decibels. In the long-distance experiment, the measurements have shown that sound attenuation owing to ground effects can be predicted with estimated average impedance values.

In reference 27, Piercy, Donato, and Embleton used estimated values of ground impedance to predict ground effects and compared them with the measurements obtained by Parkin and Scholes (refs. 25 and 26). Piercy et al. referred to the various components of the wave field by name. The direct wave, the reflected wave, and the surface wave are defined the same way as in the present paper. The ground wave, however, refers to a term in the asymptotic solution proportional to r^{-2} . The computed results agree very well with the data of Parkin and Scholes. The comparison shows that the direct-wave, the reflectedwave, and the ground-wave components are of equal importance for sound propagation over short distances. At longer distances, the high-frequency sound transmission is dominated by the combination of the direct- and reflected-wave components, and the received sound in low frequencies is dominated by the surface-wave component. Reference 30 provides an important verification for the correctness of the advanced theoretical work, and it stresses the importance of the surface-wave component for sound transmission in the lower frequencies.

The dominance of the direct- and reflected-wave combination at long distances leads to an interesting conclusion. The excess ground attenuation in this region is a simple function of the magnitude of the complex-valued acoustic impedance and the elevation angle of the sound source. In references 27, 30, and 32 this relation was considered as a way to estimate the ground impedance by means of the excess ground attenuation measurements. However, an explicit explanation was not given for the relative decline of the ground-wave term at long distances. An analysis of the relative importance of the plane wave approximation is given in the appendix.

The previously mentioned experimental investigations represent a sequence of natural development in which the ground effects are observed at increasing levels of complexity and precision. Furthermore, the comparison between theory and experiment has established that the theories can accurately describe the physical properties of ground effects on sound propagation. Therefore, computer-based prediction procedure for ground effects can be established on a solid foundation.

GROUND IMPEDANCE

Methods of Measuring Impedance

One key question relating to the study of sound propagation over ground which has not yet been satisfactorily answered is that of the best method of measuring the ground impedance. Although a number of different approaches to measuring the impedance of the ground have been investigated, none has emerged as being clearly superior under all, or even most, of the commonly occurring measurement conditions.

Impedance tube. - Of the various methods which have been tried, the most straightforward is the ordinary impedance-tube method. In this approach, the impedance tube is placed vertically on the ground (sometimes the tube is driven several inches into the ground to provide a better seal between the end of the tube and the ground), and the measurements are made in the usual way. This technique has been applied to a sand surface by Dickinson and Doak (ref. 32) and to grassland by Embleton et al. (ref. 2).

The main advantages of the impedance-tube method are the availability of this instrument and the simplicity of its operation. On the other hand, although the impedance tube would appear to be suitable for terrain such as grass, sand, and smooth soil, its usefulness in the case of surfaces such as gravel or rough soil, as well as any surface covered by a dense layer of vegetation, is doubtful owing to possible difficulties in providing a proper seal between the surface and the end of the tube. An additional drawback, which is characteristic of interference techniques in general, is that the impedance-tube method requires accurate measurements to be made of the distance from the ground to the first interference minimum of the standing-wave pattern in the tube, with the required accuracy increasing with frequency. This requirement places an upper bound on the frequency range for which this method is practicable.

Free field.- In order to avoid some of the difficulties involved in applying the impedance-tube method to real ground surfaces, Dickinson and Doak (ref. 32) have proposed an alternate approach. which they call the free-field method. The basic experimental setup consists of a sound source positioned above the ground together with a microphone which is allowed to move along a vertical axis between the source and the ground. By probing the interference pattern existing between the source and the ground, the ground impedance can be deduced. A more detailed description of the apparatus is given in reference 32.

The main advantage of the free-field method, compared to the impedance-tube technique, is that no tube or other wave-guiding apparatus is required, and hence the problem of providing a proper seal between the tube and the ground surface is avoided. Since the free-field method is, however, an interference technique, it is subject to the same frequency limitation as is the impedance-tube method. An additional difficulty, which plagues interference techniques generally, may arise in the case of surfaces such as thick grass, rough soil, etc., in which there is no obvious well-defined ground surface to use as a reference for measurements. In such cases, it is necessary to define a ground-surface reference level to be used as a basis for impedance measurements. All subsequent acoustic measurements, such as source and receiver heights, should then be based on this reference level.

Inclined track. - A variant of the free-field method, known as the inclined-track method, has been used with some success by Embleton et al. (See ref. 2.) With this technique, impedance measurements can be made at any angle of incidence, instead of only at vertical incidence as with the free-field method. The inclined-track method thus provides a means of testing the hypothesis that the ground impedance is independent of incidence angle.

Since the inclined-track method is also an interference technique, it is subject to the same limitations mentioned previously in connection with the impedance-tube and free-field methods, namely, those arising from the requirements for a very accurately defined geometry and very accurate measurements, though these restrictions become progressively less severe in changing the angle from normal to grazing incidence. In order to avoid these difficulties, a number of alternate techniques for measuring ground impedance have been proposed. Among these are what might be called curve-fitting techniques (ref. 16, sec. VII; ref. 33, p. 115; and ref. 34), the basic idea of which is to deduce the ground impedance by fitting calculated curves, obtained by inserting assumed values of the impedance into the appropriate propagation theory, to the obtained measurements of the sound field. Since this process involves, of necessity, a certain amount of trial and error, these methods are, from a strictly computational point of view, not as satisfying as the more systematic interference techniques. An additional drawback of this type of approach is that the experimenter frequently needs an initial estimate of the ground impedance to be sure that extraneous effects, such as the surface wave, are not contaminating the measurements.

Direct pressure-velocity measurements. - Since specific acoustic impedance is, by definition, the complex ratio of sound pressure to particle velocity, a direct measurement of these two parameters would seem to be an obvious approach to impedance measurement. However, the only small microphone with a response propor ional to velocity is a hot-wire device which is fragile and highly non-linear. Circuitry which linearizes the response also alters the phase so that measurement by this method becomes very complicated. This method has been used in the laboratory, usually to measure the impedance of apertures at very high sound pressures (see ref. 35), but it does not appear promising for outdoor measurements.

Pressure-gradient measurements .- Although interferometric measurements are normally made at nodal and antinodal points for convenience, the surface impedance can actually be determined from amplitude and phase measurements at other points. Mechel (ref. 36) has outlined a method which appears to have some advantages for on-site measurements. By comparing the sound pressures and their phases at two small microphones, which are closely spaced on an axis normal to the surface, Mechel has shown that the real component of the acoustic admittance is proportional to the gradient of phase change with height and that the imaginary component is proportional to the gradient of the sound pressure level. The gradient is directly proportional to the surface admittance if the measurements are made very hearly at the surface, but the admittance can be calculated from gradient measurements which are made some distance away. If the microphones are too close to the surface, their reflections alter the measured value of surface impedance, so that this method will become more useful as very small microphones become available and the accuracy of phase measurements is improved. If the gradient is not measured very near the surface, the best accuracy is obtained from measurements which are made near a nodal point where the phase gradient is large; hence different measuring positions may be needed for different frequencies.

Calculation from flight noise data .- In aircraft flyover noise measurements, acoustic data are often recorded simultaneously using a microphone which is mounted above the ground and a microphone which is flush-mounted on the ground plane. Ground-impedance information can be extracted from the ground interference pattern in the acoustic spectrum received by the raised microphone. At a given interference minimum or maximum, the magnitude of the reflection coefficient at the ground surface can be determined by the ratio of the measured sound pressure amplitude to the expected sound pressure amplitude at free-field conditions. The frequency of the interference minimum or maximum is determined by the path difference and the phase shift factor of the reflection coefficient. Since the difference in geometrical path is known, the phase factor of the reflection coefficient can be computed. The ground-surface impedance can be computed from the reflection coefficient provided that the angle of incidence is not near the zone of grazing incidence such that ray acoustic approximations are valid. In order to get meaningful impedance data, accurate position data of the source must be available and the noise data must be analyzed on a narrow-band basis since one-third-octave spectra partly conceal the minimums and maximums. This method cannot reveal the entire function of impedance as a function of frequency. The effective value of ground impedance calculated from a particular minimum may be presumed to be valid only near the frequency where it is measured.

Ground-Impedance Data

Measurements of acoustic impedance of the ground can be traced back to the early work by Nyborg et al. (ref. 37). However, the first systematic investigation of ground-impedance measurements appears to be the work by Dickinson and Doak (ref. 52). Many types of ground surfaces were examined in their work. These included sand, natural and tilled soil, short grass, chip granite, and some others. Some of the typical data are given in figure 2. One of the most interesting observations was the effect of moisture on the acoustic impedance

of soil. Perfectly dry sand gave the highest impedance, whereas the lowest impedance came when 6 to 10 percent of moisture by weight was added. Larger amounts of moisture increased the impedance. Dickinson speculated that small amounts of moisture caused grains in the soil to stick together, thereby creating larger pores, but with larger amounts of water the pores were flooded.

A substantial amount of ground-impedance data is given by Embleton, Piercy, and Olson (ref. 2). The significance of their data is that several methods have been used in the measurements. Cross-referencing among different sets of data can provide a better understanding of the variability of ground-impedance measurements. Furthermore, this data group has established a trend for the dependence of impedance as a function of frequency over an extended range. Other data of ground impedance are given by Thomasson (ref. 16) and Lanter (ref. 34). Some data on the acoustic properties of snow are also available from Tillotson (ref. 38).

A set of data for a crushed stone field filled to a depth of approximately 38 cm over a well-drained subsurface is given by R. H. Urban in unpublished correspondence to the A-21 Committee for Aircraft Noise Measurement of the Society of Automotive Engineers. Only the magnitude of the acoustic admittance is given. This set of data has been converted to the magnitude of impedance as a function of frequency and shown in figure 3. In the same figure, the data for chipped granite obtained by Dickinson et al. (ref. 32) and the data for grassland given by Embleton et al. (ref. 2) are given for comparison.

Another source of ground-impedance data comes from the indirect experiments of Delany and Bazley (ref. 28). In this work, the acoustic impedance of porous media is obtained as a function of frequency and flow resistence of the porous acoustic material. By assuming that the soil is a typical porous medium, the acoustic impedance of soil as a function of frequency can be estimated. Delany and Bazley (ref. 28) used this technique to compare theoretical calculations of ground effects with measured acoustic data, and fair agreement was obtained. Some comparison of the estimated ground impedance using this method with the data reported by Embleton, Piercy, and Olson is given in figure 4. In a recent study, Chessell (ref. 39) computed values of ground impedance by using the same model as Delany and Bazley but with a higher value of air resistance. A much better agreement with data was obtained. The comparison is shown also in figure 4.

The activities in ground-impedance measurement so far have generated sufficient data to establish trends for expected values of ground impedance. However, the data are insufficient for rating all surfaces commonly encountered in aircraft noise measurement. For example, none of the ground conditions in the previously mentioned data group can adequately match the semidesert country of the western United States where many flyover tests are performed. Most ground-impedance data collected so far tend to follow the same trend as those summarized by Embleton et al. (ref. 2). In this type of data, both the real and imaginary components of the impedance are decreasing functions of frequency. In contrast, the data obtained by Dickinson show that the real component of the impedance remains approximately constant within the frequency range of 250 to 1000 Hz.

One recognized difficulty is that existing methods of ground-impedance measurement produce data with a large amount of scatter. Part of the data scatter is a result of natural inhomogeneity of the ground-surface properties. Better methods for ground-impedance measurement are therefore needed. Theoretical modeling of ground impedance can also melp in understanding the behavior of ground impedance. In addition to the studies by Delany and Bazley (ref. 28) and Chessell (ref. 39), the recent study by Donato (ref. 40) contains theoretical results comparable to the measured data. The development of theoretical models of ground impedance is important since it may lead to alternate methods of ground-impedance measurement.

STANDARD PRACTICE IN CORRECTING FOR GROUND EFFECTS

Most current standard methods for the correction and prediction of ground effects are empirical. In the area of long-distance propagation, simple methods of prediction are commonly used. (See ref. !.) Sound attenuation is given as a function of elevation angle in these methods. However, some confusion exists regarding the range of elevation angle within which ground effects are important. According to recent theoretical results, the observed phenomena can be described analytically in a straightforward manner. The dependence of ground effects on elevation angle and ground impedance can be computed by using equations in the recommended prediction method and in the appendix. Much attention is given to methods of data correction for ground-based engine-noise and flyover-noise measurements. In this section, common practices in the latter category are discussed.

The objective of data correction is to recover the free-field spectrum from measurements with ground effects. The two possible approaches are either to minimize the acoustic problems associated with reflection or to correct for the ground effects during data processing.

Modification of Physical Environment

Flush-mounted microphones. - The first dip caused by multipath interference over a rigid surface occurs at a frequency given by

$$f_0 = \frac{r_1 c}{4hz}$$

where r_1 is the source-to-microphone distance, c is the speed of sound, and h and z are the heights of source and receiver, respectively. By reducing the microphone height, f_0 can be made as high as necessary. For a distance of 60 m, for example, and a sound source 2 m above the ground, f_0 will be above 10 kHz for microphone heights below 25 cm. Since the direct and the reflected waves will arrive at the microphone approximately in phase, the sound pressure is double the free-field condition. Therefore, 6 dB should be subtracted from all frequency bands for free-field equivalence.

Flush-mounted microphones are now being extensively used for outdoor acoustic measurements, especially for engine static tests. Owing to the effects of ground and wind and temperature gradients near the ground, precautions should be taken to guard against high-frequency losses in this configuration. Data above 1 kHz should be compared with equivalent bands as measured on a microphone at least 1 m above the surface. It is common practice to combine the low-frequency data from flush-mounted microphones with high-frequency data from the raised microphones, thereby reducing the flush-mounted-microphone data by 6 dB and the raised-microphone data by about 2 dB to obtain a composite free-field sound pressure level spectrum.

The flush-mounted microphone should be used only over hard surfaces since the phase shift for reflection from a porous surface can introduce serious frequency distortion in the spectrum. For aircraft flyover measurements, good results have been obtained by facing a sheet of plywood approximately 1.25 m by 1.25 m in dimension with aluminum and mounting the microphone in a hole drilled in the center. Furthermore, flush-mounted microphones should not be used for sideline noise measurements since ground effects, such as the surface wave, can introduce errors in the low-frequency data.

Raised microphones.— It is practical also to arrange microphones high above the ground surface so that the first interference dip falls below the frequency range of common interest. For example, a separation distance of 20 m combined with source and microphone heights at 6 m will bring the first interference dip in the spectrum to about 50 Hz. Presumably the one-third-octave band at 50 Hz is not considered significant, and the higher order interference patterns are narrow compared with the one-third-octave bandwidth and are thus less influential on the measured data. This method is susceptible to errors caused by environmental factors and appears to be less accurate than the flush-mounted-microphone concept.

Gravel and similar partly controlled impedance surfaces.— Many aerospace organizations use gravel or sand areas for static acoustic measurements to avoid the cost of large concrete pads. This practice gives reasonable repeatability of measurements for microphones which are at least a meter from the surface. The acoustic impedance of gravel test areas has been found to be relatively constant provided that they have sufficient thickness, are well drained, and remain above the water table. Since the engine static test facilities have a high rate of utilization, the gravel-filled area is sometimes acoustically calibrated for ground effects. The effects of the wind vector are often included in the calibration.

Numerical Ground-Effects Correction

Data correction with known ground impedance.— If reliable data of ground impedance are known, ground effects in measured data can be corrected by means of analytical calculations. Figure 5 shows a comparison between a one-third-octave band spectrum of a jet engine on static test as recorded over a grave) surface with a microphone 75 m away and as recorded from a balloon the same distance above the source. The first microphone and the sound source were 4 m above the gravel surface. The presumed ground reflection characteristic is

determined by the difference between the two spectral curves and is shown in the lower half of figure 5 as a solid-line curve. The points are the calculated values of the ground reflection characteristics using an impedance value taken from Dickinson's measurement for a broken tarmacadam surface. Even with the rough impedance estimate, there is a very good agreement with the measured data. Other discussions of data correction with known acoustic properties of the ground are given in references 41 and 42.

Miscellaneous techniques. - Data which show prominent dips in lower frequency bands are frequently corrected by the trained eye and hand of the test engineer. This correction can be fairly successful for data obtained over hard surfaces if the test engineer understands the theoretical background and if the expected free-field spectrum is guided by, for example, model test data obtained in an anechoic chamber. In common practice the peak in the lowest frequencies will be lowered by 3 to 4 dB, a smooth line will be drawn through the first dip, and the high-frequency bands will be lowered 1 or 2 dB with peaks and troughs smoothed out. This procedure will obviously be incorrect in cases where a strong tone is present. Therefore, the test engineer should also be familiar with the characteristics of the sound source so as to distinguish which frequency bands legitimately should contain interference corrections and which ones may include real tones. He is also responsible for recognizing erratic behavior of the data, such as electronic noise sources.

Computer algorithms performing the same sequence of operation have appeared in recent years. This procedure has been used by research organizations where high-volume data processing is required. The computer-based method can utilize the theoretical results for ground effects in a more positive manner than the manual method.

Note also that correction by visual smoothing can lead to serious error for data taken at long distances over a surface with finite impedance. The high-frequency acoustic signal can be systematically reduced through the action of ground effects. Consequently the ground-effects correction may require an increase from the measured level instead of the more common subtraction of 1 or 2 dB.

Analytic correction methods.- A number of computer-based methods for the correction of ground effects exist. Each method is designed for use at a specific facility. However, thewe methods are definitely applicable at other facilities of similar type. The method given by Miles (refs. 43 and 44) is designed for an asphalt-surfaced acoustic test area where the typical distance of measurement is 20 m and the source and microphone heights are approximately 4 m above the ground. This method is an iterative scheme where a model freefield spectrum is postulated at the beginning of the iteration. Ground effects are added to this spectrum and the result is compared with the measured data. A special error function is chosen to measure the agreement. If the error limit is exceeded, the parameters for the model spectrum and the ground effects are modified and a second iteration will be completed. Owing to the relatively short distance of measurement, a multiple-source location problem is often encountered for large test objects at this facility. This method has been found to provide satisfactory results for ground-effects correction under such conditions.

Another approach is often adopted for ground-effects correction in gravel-filled test areas with typical source-to-microphone distances of 40 m to 60 m. The facility is calibrated acoustically to determine some typical parameters of ground impedance and expected ground effects on acoustic measurements. The computer program initiates a search for the frequency and magnitude of the first interference dip and then applies the appropriate ground-effects correction for the entire sound pressure level spectrum according to the information obtained in the acoustic calibration of the facility. These methods are also found to be successful.

_epstral technique.- Another useful technique for ground-effects correction was developed by Miles et al. (ref. 45). For acoustic measurements at short range with the sound source and the microphone high above a hard surface, the maximums and minimums of the interference pattern occur at regular frequency intervals. Furthermore, the first interference dip often occurs at a very low frequency which lies below the range of practical interest so that it may be ignored. A Fourier transformation of the logarithm of the spectrum results in a cepstral function in which interference maximums and minimums are represented as a single spike. The location of this spike is determined by the frequency span between two consecutive maximums or minimums. By removing this spike from the cepstral function and taking an inverse Fourier transformation, the result is a logarithm spectrum without ground effects. Note that this technique is applicable only for acoustic measurements over a hard surface where the surface reflection does not introduce any significant phase shift into the acoustic signal.

The importance of ground effects has been recognized, and offective measures have been taken in major test facilities for the improvement of physical environments for outdoor acoustic measurements. In the area of data analysis, the methods now in existence show a trend of continued improvement. In particular, successful computer algorithms of ground-effects correction are now available for specific conditions of test environment. The next logical step is to implement fully the advanced theoretical results in a computerized scheme.

RECOMMENDED PREDICTION METHOD

Specific formulas, obtained from recent theory, are suggested herein for the purpose of making predictions of the acoustic field in real experimental situations. These formulas are, as previously mentioned, based on the approximate results of Chien and Soroka (ref. 11) and supplemented by the exact integral solution of Thomasson (ref. 12). The physical model of sound propagation over ground which forms the basis of these theories has been discussed in the "Introduction." The essential features of this model are the assumptions of a flat, locally reacting ground surface and a uniform atmosphere, and to the extent that these assumptions are justified for the particular experiment in question, the formulas can be expected to yield accurate predictions. Note that these formulas were derived for the case of time-harmonic waves radiated by a point source and hence can be applied directly only to this case. However since these formulas represent Green's function solutions, they can be used to treat more general situations by means of superposition. For example, the solution for the case of a distributed source can be obtained by integrating

the point-source solution, weighted by the appropriate source term, over the source volume. A similar approach can be used to treat the case in which the source has a continuous frequency spectrum.

The geometry of the situation is shown in figure 1. A point time-harmonic source S is located at a height h above an impedance boundary which is taken to be coincident with the x,y plane. The distances from S and S' (the image source) to the receiver P are denoted by r_1 and r_2 , respectively; r is the horizontal distance between S and R; and θ is the angle of incidence.

In mathematical terms, the problem of interest involves the solution of the equation

$$(\nabla^2 + k^2)\phi = \delta(x)\delta(y)\delta(z - h) \tag{1}$$

in the region z > 0 of x,y,z space subject to the boundary condition

$$\frac{\partial \phi}{\partial z} + ik \psi \phi = 0 \tag{2}$$

on z=0. Here ϕ is the (complex) acoustic velocity potential and $k=\omega/c$, where ω is the circular frequency (assumed positive) and c is the speed of sound. Also v is the specific boundary admittance, which is assumed constant, and is written in the form $v=v_1+iv_2$ where $v_1\geq 0$. In addition to the boundary condition given by equation (2), a radiation condition corresponding to outgoing waves is understood to be imposed at infinity but is not explicitly indicated here. A time-harmonic factor $e^{-i\omega t}$, common to both the source and the wave field, is also understood. The complex acoustic pressure amplitude p is given in terms of the velocity potential by the relation $p=-i\omega\rho_0\phi$, where ρ_0 is the unperturbed density of the acoustic medium.

As noted previously, the various approximate solutions obtained by Chien and Soroka cover a wide range of practical situations, provided only that $kr_2 >> 1$. Within this rather mild constraint, several subcases appear naturally and are defined below along with the corresponding approximate solutions (written in the present notation), which are recommended herein for prediction purposes.

(1) $|v| \ll 1$. In this case, the appropriate expression for the wave field is obtained from equation (25) of reference 11, and can be written in the form

$$\phi = -\frac{1}{4\pi} \left\{ \frac{e^{ikr_1}}{r_1} + \left[\Gamma + (1 - \Gamma) F(\sigma)\right] \frac{e^{ikr_2}}{r_2} \right\}$$
 (3)

where Γ is the plane wave reflection coefficient, defined by

$$\Gamma = \frac{\cos \theta - v}{\cos \theta + v} \tag{4}$$

The function F is given by

$$F(\sigma) = 1 - \pi^{1/2} \sigma e^{\sigma^2} \operatorname{erfc}(\sigma)$$
 (5)

where o is defined by

$$\sigma = \left(\frac{kr_2}{2i}\right)^{1/2} (\cos \theta + \nu) \tag{6}$$

In equation (6) the principal branch of the square root is understood; that is, the square root of any complex number ζ is defined by

$$\zeta^{1/2} = |\zeta|^{1/2} \exp\left(\frac{1}{2} i \arg \zeta\right)$$

where $-\pi < \arg \zeta \le \pi$. Since $\frac{-\pi}{2} < \arg v \le \frac{\pi}{2}$, the argument of σ lies in the

range $\frac{-3\pi}{4} \le \arg \sigma \le \frac{\pi}{4}$. The solution given by equations (3) to (6) has essen-

tially the same form as those of Ingard (ref. 8, eq. (13)) and Lawhead and Rudnick (ref. 9, eq. (21)). These authors did not, however, make clear the restrictions on the validity of their solutions.

The power-series expansion

$$erfc(\sigma) = 1 - \frac{2}{\sqrt{\pi}} e^{-\sigma^2} \sum_{n=0}^{\infty} \frac{2^n}{(2n+1)!!} \sigma^{2n+1}$$
 (7)

of the complementary error function when inserted into equation (5) yields the expansion

$$F(\sigma) = 1 - \sqrt{\pi}\sigma e^{\sigma^2} + 2\sigma^2 \sum_{n=0}^{\infty} \frac{(2\sigma^2)^n}{(2n+1)!!}$$
 (8)

which is convenient for calculations when $|\sigma| \ll 1$. Recall that

$$(2n + 1)!! \equiv 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n + 1)$$

Similarly, the asymptotic expansion

erfc(
$$\sigma$$
) $\approx 2U(-\text{Re }\sigma) + \frac{e^{-\sigma^2}}{\sqrt{\pi}\sigma} \left[1 - \frac{1}{2\sigma^2} + \frac{3}{(2\sigma^2)^2} - \dots \right]$ (9)

leads to the result

$$F(\sigma) \approx -2\sqrt{\pi}U(-\text{Re }\sigma)\sigma e^{\sigma^2} + \frac{1}{2\sigma^2} - \frac{3}{(2\sigma^2)^2} + \dots$$
 (10)

which is useful when $|\sigma| >> 1$. Here U denotes the unit step function, which is defined by

$$U(s) = \begin{cases} 1 & (s > 0) \\ \frac{1}{2} & (s = 0) \\ 0 & (s < 0) \end{cases}$$

The surface-wave term is easily verified to be implicit in the approximations given by equations (8) and (10) by simply truncating both expansions after two terms and inserting the result into equation (3). For the special case in

which $\theta = \frac{\pi}{2}$ (i.e., both source and receiver on the boundary), this procedure yields

$$\phi = -\frac{e^{i\mathbf{k}\mathbf{r}}}{2\pi\mathbf{r}} + \frac{1}{2}\nu\left(\frac{\mathbf{k}}{2\pi\mathbf{r}}\right)^{1/2} e^{i\left[\left(1-\frac{1}{2}\nu^2\right)\mathbf{k}\mathbf{r}-\frac{\pi}{4}\right]}$$
(11)

when $|\sigma| \ll 1$, and

$$\phi = -\frac{ie^{ikr}}{2\pi v^2 kr^2} + U(-Re \sigma)v\left(\frac{k}{2\pi r}\right)^{1/2} e^{i\left[\left(1-\frac{1}{2}v^2\right)kr-\frac{\pi}{4}\right]}$$
(12)

when $|\sigma| >> 1$. The second terms on the right-hand sides of equations (11) and (12) can be identified as surface-wave terms. Note that the surface-wave term in equation (12) appears only when Re $\sigma \le 0$, or what is equivalent, when

$$-\frac{\pi}{2} \le \arg v \le \frac{\pi}{4}$$
.

Note that, although the original derivation of equation (25) of Chien and Soroka (ref. 11), or, equivalently, equation (3) of the present paper, required the additional assumptions that $kr \gg 1$ and $k(h+z)^2/r \ll 1$, Chien and Soroka (ref. 11, sec. 6) showed that these additional conditions can be relaxed.

(2) $|v| \ge 1$ (i.e., |v| is of order 1 or greater). In this case the appropriate expression for ϕ is given by equation (12) of reference 11, which, after some manipulation, can be written

$$\phi = \Phi \qquad (v_2 \ge -f(v_1, \theta)) \qquad (13a)$$

$$\phi = \phi + \Psi \qquad (v_2 < -f(v_1, \theta)) \qquad (13b)$$

where

$$\Phi = \frac{1}{4\pi} \left\{ \frac{e^{ikr_1}}{r_1} + \left[\Gamma + \frac{2i\nu}{kr_2} \frac{1 + \nu \cos \theta}{(\cos \theta + \nu)^3} \right] \frac{e^{i\kappa r_2}}{r_2} \right\}$$
(14)

$$\Psi = \frac{1}{2} \text{ kve}^{-i\nu k(h+z)} H_0^{(1)} \left[(1 - v^2)^{1/2} \text{kr} \right]$$
 (15)

and

$$f(v_1,\theta) = \csc \theta(\cos \theta + v_1)(1 + v_1 \cos \theta)(1 + 2v_1 \cos \theta + v_1^2)^{-1/2}$$
 (16)

Here Ψ is the surface-wave term, and $H_0^{(1)}$ denotes the Hankel function. The function $f(\nu_1,\theta)$ for selected values of θ is plotted in figure 6. The surface-wave term in equation (13b) turns out to be negligible at all except near-grazing angles of incidence. To see this, note that, from figure 1, $h\leftrightarrow z=r_2\cos\theta$; also, as a consequence of the condition on ν for the existence of the surface wave and the assumption that $kr_2 \gg 1$, note that $-\nu_2 \ge 1$ whenever the surface wave exists. Hence

which, since $kr_2 >> 1$, is negligible whenever $\cos \theta \approx 1$. Note also that the condition $|v| \gtrsim 1$ of this subsection can be relaxed at all except near-

grazing angles of incidence. This follows from the fact that the actual condition for the validity of equation (12) of reference 11 is $|\cos\theta+\nu|^2kr_2>>1$. Therefore, provided that $kr_2>>1$, the assumption $|\nu|^2$ 1 is needed only when $\cos\theta<<1$.

Up to this point, the results of this section require for their validity that $kr_2 >> 1$. In order to treat the case in which $kr_2 \leq 1$, it is necessary

to resort to numerical evaluation of the exact integral expression for the wave field. For this purpose, the integral expression of Thomasson (ref. 12, sec. V) appears to be most suitable. This solution can also be used as a check on the approximate results given previously in this section.

In the present notation, Thomasson's solution car be written

$$\phi = -\frac{1}{4\pi} \left(\frac{e^{ikr_1}}{r_1} + \frac{e^{ikr_2}}{r_2} \right) + H \tag{17}$$

where

$$H = F \qquad (v_2 \ge -f(v_1, \theta)) \qquad (18a)$$

$$H = F + G$$
 $(v_2 < -f(v_1, \theta))$ (18b)

Here $f(v_1,\theta)$ is given by equation (16) and

$$F = \frac{kV}{2\pi} e^{ikr_2}I$$
 (19)

where

$$I = \int_{0}^{\infty} D^{-1/2} e^{-kr_2 t} dt$$
 (20)

and

$$D = (\cos \theta + v)^2 + 2i(1 + v \cos \theta)t - t^2$$
 (21)

In calculating the square root of D, the principal branch, as defined previously, is to be used except when $v_2 < -f(v_1, \theta)$ and $t > t_1$, where

$$t_1 = -v_2(\cos \theta + v_1)(1 + v_1 \cos \theta)^{-1}$$

in which case the negative of the principal value is to be used. The quantity G in equation (18b) is the surface-wave term; that is,

$$G = \frac{1}{2} kve^{-ivk(h+z)}H_0^{(1)}[(1-v^2)^{1/2}kr]$$

Note that the surface-wave term and the condition for its existence are the same as given by equations (13b) and (15).

PROBLEMS IN APPLICATION

The preceding theoretical development is based on a set of idealized conditions which are not always met in practical applications. Some common problems in this direction are now discussed.

Finite Source

The effects of finite-source dimension have been analyzed in detail by Thomas (ref. 46). In a fixed geometrical configuration where both the source and the microphone positions are high above the ground surface, ground effects on sound propagation are a result of phase differences between the direct and the reflected waves caused by ground reflection and path length differences. The maximum destructive interference occurs at a specific frequency where the total phase mismatch is 180°. However, such frequency is sharply defined only if the sound is emitted from a point source. In practice, a single jet engine may have a diameter greater than 2.5 m.

Consider an engine roise test at static condition. The center line of the engine and the microphone are both positioned at 5.5 m above the ground with a horizontal separation distance of 60 m. Furthermore, assume that the ground surface is paved with concrete. In this configuration, the path difference computed from the center of the engine to the position of the microphone is approximately 1 m, and the corresponding frequency for maximum cancellation is approximately 174 Hz. If the source positions along the lip line at the engine intake are considered, the maximum cancellation frequency ranges from 141 Hz for points farthest from the ground to 224 Hz for points nearest to the ground. Hence, the acoustic energy emitted near the lip line will have a range of cancellation frequencies extending over at least eight-tenths of an octave. The measured acoustic spectrum will show a broad but shallow dip in this frequency range, rather than a sharp minimum.

Filter Bandwidth

The modification of the interference pattern by finite filter bandwidths has been analyzed by Howes (ref. 47) and discussed in reference 1. The general effect of the one-third-octave filters commonly used is to average out peaks and dips within bands above 1000 Hz and to broaden the lower ones similar to the effect of extended source dimension. The filters do not reduce much of the depth of the first interference dip since the width of this dip is larger than the filter bandwidth.

Observe that the filtering procedure may change the measurements but has no effect on the physical phenomenon. Thus is there is a prominent tone at a single frequency in a high one-third-octave band, it may be increased or canceled by multipath interference. The average ground-effects correction for the entire band would not be applicable for recovering the correct value of this tone.

Partial Coherence

For noise emission with a broadband spectrum, such as jet noise, the origin of noise is often random in nature. The idea of phase cancellation should be treated with special care. The amplitude of the combined direct and reflected waves depends on the maintenance of coherent phase relations between wave components arriving at the point of measurement by different paths. The phase relation can also be lost through the effect of random perturbations along the nath of propagation or as a result of reflection on a nonhomogeneous ground surface. The effect of coherence on sound pressure level can be shown in a simple analysis. A coherence coefficient can be defined as

$$C(\omega) = Q(\omega) / [S_0(\omega)S_r(\omega)]^{1/2}$$
(22)

where $S_0(\omega)$ and $S_r(\omega)$ are the sound power spectral density of the direct and the reflected signals, respectively, and $Q(\omega)$ is the magnitude of the cross-spectral density function between the direct and the reflected waves. (Note that this function is not always measurable in the configuration of ground reflection. It is well defined, however, when both the direct and reflected acoustic signals are known.) The coherence coefficient $C(\omega)$ can be considered as the fraction of initial acoustic energy in which phase relation is maintained throughout the propagation process. The balance of the acoustic energy is distributed in waves of the same frequency with random phase relations.

At the point of recombination of the direct- and reflected-wave components, the coherent portion of the wave will be summed according to the applitude and phase of the two components, whereas the incoherent portion of the wave energy will be summed according to the values of the acoustic intensity. This leads to the equation

$$\frac{p^2}{p0^2} = C(\omega) [1 + 2R \cos(\alpha + kd) + R^2] + [1 - C(\omega)](1 + R^2)$$
 (23)

where p represents the combined sound pressure, p_0 is the expected free-field sound pressure at the microphone position, d is the length difference between the reflected path and the direct path, and R and α are the amplitude and phase of the reflection coefficient, that is,

$$\Gamma(\omega) = R(\omega)e^{i\alpha(\omega)}$$
 (24)

where $\Gamma(\omega)$ is defined in equation (4). The values of R and α are frequency dependent. Since the coherence coefficient is not always a measurable function, some assumptions may be necessary for practical applications. A reasonable choice is a Gaussian distribution

$$C(\omega) = e^{-(a\omega)^2}$$
 (25)

where a is a real and positive constant.

At the frequency of maximum destructuve interference, the direct- and reflected-wave components are 180° out of phase at recombination. Equation (23) can be simplified to give the relative sound pressure level in reference to the expected free-field sound pressure level (SPL) at the microphone position

$$\Delta SPL = 10 \log_{10} \left\{ C(\omega)(1 - R)^2 + \left[1 - C(\omega) \right] (1 + R^2) \right\}$$
 (26)

Calculated values of this interference dip for different values of $C(\omega)$ and R are given in figure 7. Note that the presence of a small fraction of incoherent wave can make drastic changes in the value of the interference attenuation. For example, with a reflection coefficient of 0.90, the magnitude of the interference dip reduces from 20 dB for a wave with perfect coherence to merely 7.9 dB for a wave with a coherence coefficient of 0.95.

In the case of jet noise, sound emitted in different directions will not be perfectly coherent. According to experimental indications, the coherence factor (ref. 48, figs. 20 and 21) is

$$C(\omega) = \cos^4 f$$
 (27)

where β is the angle between the direct and the reflected ray paths at the source. For an emission angle difference of 10° , which is common for typical engine static test configurations, the coherence factor is approximately 0.94. Furthermore, jet noise is coherent only over a finite length of time. The typical coherence time scale for a large jet engine is of the order of 10 to 15 msec. For a propagation path difference of more than a few meters, the time incoherence of the acoustic signal must be taken into consideration. Such conditions may be encountered for flyover measurements with microphones positioned high above the ground.

Under conditions of long-range propagation, several additional mechanisms may become significant in loss of signal coherence. Among the most important of these are the effects of atmospheric turbulence and the effects of random ground roughness. An analysis of the former has been given by Ingard and Maling (ref. 49); however, that analysis considered only single scattering and hence is valid only for relatively limited propagation distances. With regard to the latter, mathematical techniques which include multiple-scattering effects, and which are therefore applicable to long-range propagation, have only recently been developed. (See, e.g., refs. 50 to 52.) No attempt has yet been made to apply these techniques to real problems involving propagation of sound over ground. It thus appears that predictions of coherence loss due to these mechanisms must remain, at least for now, largely empirical.

An additional effect of atmospheric turbulence, which is not related to the presence of a nearby reflecting surface, is manifested in the appearance of random fluctuations in the sound pressure level. Observations indicate that these fluctuations increase initially with propagation distance, but that at longer distances they approach a limiting, or saturation, value. (See, e.g., ref. 3, sec. IIIC.) This saturation phenomenon, which is a general feature of

wave propagation in a random medium and which is not amenable to a single-scattering analysis, has been the subject of a number of recent theoretical investigations. (See, e.g., ref. 53 and references cited therein.)

Wind Gradients

In the presence of wind, the analysis of ground effects becomes very difficult from a theoretical point of view. However, in geometrical configurations typical of ground-based engine noise measurements, the effects of a small wind vector can be included by means of simplified analysis. The phase difference frequency is determined by the phase shift caused by the ground reflection and the propagation path difference. In the presence of a wind vector, the difference in average propagation velocity between the direct and the reflected paths will produce an additional segment of path difference. For a typical boundary-layer profile, this additional path difference can be given as

$$d_1 = 0.3 \text{rv/c}$$
 (28)

where r is the horizontal distance, c is the speed of sound, and v is the wind-velocity component along the direction from the source to the receiver. In equation (28), the source and the microphone are assumed to be at the same height above the ground. According to recent studies, the deviation of the frequency for maximum interference attenuation can be approximately two-thirds octave on either side of the expected peak attenuation frequency. For practical applications the effect of wind on the wave-propagation path difference should be included in the computations of ground effects.

CONCLUDING REMARKS

A unified method has been recommended for the prediction of ground effects. This method is applicable to long-range sound propagation, correction of data in engine test-stand configurations, and aircraft flyover noise measurements. The results given in this paper contain sufficient detail for the purpose of numerical calculation of ground effects.

Sufficient data are available at this time to form a band of expected values for ground impedance for common surfaces such as grassland, sand, and soil. Measurements from different sources seem to agree in general. However, there is a large variance of data scattering. In order to obtain more accurate data on ground impedance, methods of measurement should be improved. Furthermore, the measurement of ground impedance using existing methods is time consuming. Improved procedures may encourage researchers to make on-site measurements of ground impedance in conjunction with acoustic measurements.

Theoretical analysis for applications to aircraft noise prediction is adequate for the present. However, the analysis of coupled effects of ground and atmospheric perturbations on sound propagation near the ground seems to provide the possibility of a better description of the natural environment. Further research in the area of layered representation of the ground surface may also be beneficial to practical applications.

From an analytical point of view and also from evidence shown in existing experimental results, the recommended method appears to be sufficiently accurate for practical applications. However, the method should be validated before it is accepted for general industrial applications. Since improved instrumentation and facilities of data analysis are now available, an experiment for sound propagation over the ground with careful documentation of ground impedance and atmospheric conditions should be conducted.

A validation program is beneficial from another point of view. In some field measurements, the ground-impedance environment will not be known beyond a general description of the physical appearance of the ground condition. In the process of validation, experience can be gained so that a band of nominal values for various parameters can be recommended. Thus, an estimate of ground effects can be obtained in the absence of direct information concerning the environmental factor.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
December 8, 1977

APPENDIX

RELATIVE IMPORTANCE OF PLANE WAVE APPROXIMATION

A general expression for the sound pressure field near an impedance boundary can be given as

$$p = \frac{e^{ikr_1}}{4\pi r_1} + \left\{ \Gamma(\theta) + \left[1 - \Gamma(\theta)\right] F(\sigma) \right\} \frac{e^{ikr_2}}{4\pi r_2}$$
(A1)

Symbols in equation (A1) are defined the same as those in the main text. If the effect of the surface wave is not considered, then

$$[1 - \Gamma(\theta)]F(\sigma) = \frac{2i\nu}{kr_2(\cos\theta + \nu)^3}$$
(A2)

when $|kr_2v^2| >> 1$ and $v \cos \theta << 1$. For sound propagation over a long horizontal distance, the path-length difference between the direct and the reflected waves is very small. Therefore

$$r \approx r_1 \approx r_2$$
 (A3)

and the direct and the reflected waves can be combined into one term (ref. 31)

$$F_1 = \frac{e^{ikr}}{2\pi r(\cos\theta + v)} \cos\theta \tag{A4}$$

An angle θ ' can be defined such that θ ' = $\frac{\pi}{2}$ - θ . For a receiver height that

is small compared to the source height, or conversely, the angle θ ' is approximately equal to the elevation angle of the source. For small values of the angle θ '

$$\cos \theta = \sin \theta' \approx \theta' \approx \frac{kh}{kr}$$
 (A5)

By substituting equations (A2) to (A5) into equation (A1), the sound pressure field can be given in a simple form

$$p = \frac{e^{ikr}}{2\pi kr^2(\cos\theta + v)v} \left[khv + i \left(\frac{v}{\cos\theta + v} \right)^2 \right]$$
 (A6)

It is clear from equation (A6) that the combination of direct and reflected waves given by equation (A4) dominates when

 $|\mathbf{kh}v| \gg 1$ (A7)

This result leads to a number of interesting conclusions as follows:

- (1) Both the combination term and the ground-wave term (according to the definition of ref. 27) attenuate at the rate of r^{-2} . These two terms are not separable in measurements at long distances away from the sound source.
- (2) If the condition specified in equation (A7) is met, the ground-wave term can be neglected. The overall sound pressure is directly proportional to the elevation angle only when $\cos\theta << |\nu|$. (See eq. (A4).) The ground effect is negligible for $\cos\theta >> |\nu|$. Therefore, this equation provides an analytical basis for the empirical methods for estimating ground attenuation by using the elevation angle alone. For aircraft in-flight sideline noise measurements, kh is normally large and equation (A7) is satisfied. However, the ground attenuation should be considered as a function of frequency since the ground admittance is a function of frequency in most cases.
- (3) This analysis also provides a limit to the validity of the previously mentioned empirical method. It should not be applied to the low-frequency range or under conditions where the surface wave may have a strong influence.

Finally, note that Piercy et al. (ref. 31) pointed out that the ground wave is a correction of the sphericity of the wave front. Naturally, the ground-wave term becomes less important at distances far from the source since the sound field can be better approximated by a plane wave.

REFERENCES

- Putnam, Terrill W.: Review of Aircraft Noise Propagation. NASA TM X-56033, 1975.
- Embleton, T. F. W.; Piercy, J. E.; and Olson, N.: Outdoor Sound Propagation Over Ground of Finite Impedance. J. Acoust. Soc. America, vol. 59, no. 2, Feb. 1976, pp. 267-277.
- Piercy, J. E.; Embleton, T. F. W.; and Sutherland, L. C.: Review of Noise Propagation in the Atmosphere. J. Acoust. Soc. America, vol. 61, no. 6, June 1977, pp. 1403-1418.
- Pao, S. P.; and Evans, L. B.: Sound Attenuation Over Simulated Ground Cover. J. Acoust. Soc. America, vol. 49, no. 4, pt. 1, Apr. 1971, pp. 1069-1075.
- Van Moorhem, W. K.; and Shepherd, K. P.: Consideration in the Use of Ground Mounted Microphones. Proceedings - Third Interagency Symposium on University Research in Transportation Noise, Univ. of Utah, Nov. 1975, pp. 114-122.
- 6. Brekhovskikh, Leonid M. (David Lieberman, transl., Robert T. Beyer, ed.): Wave in Layered Media. Academic Press, Inc., 1960.
- 7. Morse, Philip M.; and Bolt, Richard H.: Sound Waves in Rooms. Rev. Modern Phys., vol. 16, no. 2, Apr. 1944, pp. 69-150.
- 8. Ingard, Uno: On the Reflection of a Spherical Sound Wave From an Infinite Plane. J. Acoust. Soc. America, vol. 23, no. 3, May 1951, pp. 329-335.
- Lawhead, R. B.; and Rudnick, I.: Acoustic Wave Propagation Along a Corestant Normal Impedance Boundary. J. Acoust. Soc. America, vol. 23, no. 5, Sept. 1951, pp. 546-549.
- Wenzel, Alan R.: Propagation of Waves Along an Impedance Boundary.
 J. Acoust. Soc. America, vol. 55, no. 5, May 1974, pp. 956-963.
- Chien, C. F.; and Soroka, W. W.: Sound Propagation Along an Impedance Plane. J. Sound & Vib., vol. 43, no. 1, Nov. 8, 1975, pp. 9-20.
- Thomasson, Sven-Ingvar: Reflection of Waves From a Point Source by an Impedance Boundary. J. Acoust. Soc. America, vol. 59, no. 4, Apr. 1976, pp. 780-785.
- 13. Brekhovskikh, L. M.: Surface Waves in Acoustics. Sov. Phys.-Acoust., vol. 5, no. 1, Jan.-Mar. 1959, pp. 3-12.
- 14. Baños, Alfredo, Jr.: Dipole Radiation in the Presence of a Conducting Half-Space. Pergamon Press, Inc., 1966.

- Wait, James R.: Electromagnetic Waves in Stratified Media. Second ed., Pergamon Press, Inc., c.1970.
- Thomasson, Sven-Ingvar: Sound Propagation Above a Layer With a Large Refraction Index. J. Acoust. Soc. America, vol. 61, no. 3, Mar. 1977, pp. 659-674.
- Donato, R. J.: Propagation of a Spherical Wave Near a Plane Boundary With a Complex Impedance. J. Acoust. Soc. America, vol. 60, no. 1, July 1976, pp. 34-39.
- Donato, R. J.: Spherical-Wave Reflection From a Boundary of Reactive Impedance Using a Modification of Cagniard's Method. J. Acoust. Soc. America, vol. 60, no. 5, Nov. 1976, pp. 999-1002.
- Briquet, Martine; and Filippi, Paul: Diffraction of a Spherical Wave by an Absorbing Plane. J. Acoust. Soc. America, vol. 61, no. 3, Mar. 1977, pp. 640-646.
- Van Moorhem, W. K.: Reflection of a Spherical Wave From a Plane Surface.
 J. Sound & Vib., vol. 42, no. 2, Sept. 22, 1975, pp. 201-208.
- Rudnick, Isadore: The Propagation of an Acoustic Wave Along a Boundary.
 J. Acoust. Soc. America, vol. 19, no. 2, Mar. 1947, pp. 348-356.
- Ingard, Uno: A Review of the Influence of Meteorological Conditions on Sound Propagation. J. Acoust. Soc. America, vol. 25, no. 3, May 1953, pp. 405-411.
- 23. Oleson, S. K.; and Ingard, U. K.: Field Measurements of Sound Propagation. Contract NASw-6516, Massachusetts Inst. Technol., Dec. 30, 1959.
- 24. Wiener, Francis M.; and Keast, David N.: Experimental Study of the Propagation of Sound Over Ground. J. Acoust. Soc. America, vol. 31, no. 6, June 1959, pp. 724-733.
- 25. Parkin, P. H.; and Scholes, W. E.: The Horizontal Propagation of Sound From a Jet Engine Close to the Ground, at Radlett. J. Sound & Vib., vol. 1, no. 1, Jan. 1964, pp. 1-13.
- 26. Parkin, P. H.; and Scholes, W. E.: The Horizontal Propagation of Sound From a Jet Engine Close to the Ground, at Hatfield. J. Sound & Vib., vol. 2, no. 4, Oct. 1965, pp. 353-374.
- Piercy, J. E.; Donato, R. J.; and Embleton, T. F. W.: Near-Horizontal Propagation of Sound Over Grassland. J. Acoust. Soc. America, vol. 60, suppl. no. 1, Fall 1976, p. S2.
- 28. Delany, M. E.; and Bazley, E. N.: Acoustic Properties of Fibrous Absorbent Materials. Appl. Acoust., vol. 3, no. 2, Apr. 1970, pp. 105-116.

- Delany, M. E.; and Bazley, E. N.: A Note on the Effect of Ground Absorption in the Measurement of Aircraft Noise. J. Sound & Vib., vol. 16, no. 3, June 8, 1971, pp. 315-322.
- Piercy, J. E.; Embleton, T. F. W.; and Olson, N.: Impedance of Soft Ground and Its Effect on Practical Measurements. J. Acoust. Soc. America, vol. 54, no. 1, July 1973, p. 341.
- 31. Piercy, J. E.; and Embleton, T. F. W.: Effect of the Ground on Near-Horizontal Sound Propagation. Reprint 740211, Soc. Automot. Eng., 1974.
- 32. Dickinson, P. J.; and Doak, P. E.: Measurements of the Normal Acoustic Impedance of Ground Surfaces. J. Sound & Vib., vol. 13, no. 3, Nov. 1970, pp. 309-322.
- Jonasson, H. G.: Sound Reduction by Barriers on the Ground. J. Sound & Vib., vol. 22, no. 1, May 8, 1972, pp. 113-126.
- 34. Lanter, Sean K.: A Method for Determining Acoustic Impedance of Ground Surfaces. M.S. Thesis, Univ. of Utah, 1977.
- 35. Ingard, Uno; and Ising, Hartmut: Acoustic Nonlinearity of an Orifice.
 J. Acoust. Soc. America, vol. 32, no. 1, July 1967, pp. 6-17.
- 36. Mechel, Fr.: New Method of Impedance Measurement (Neues Impedanzmessver-fahren). Reports of the 6th International Congress on Acoustics, [Volume] V, Y. Kohasi, ed., Elsevier Pub. Co., 1968, pp. H-217 H-220.
- 37. Nyborg, W. L.; Rudnick, I.; and Schilling, H. K.: Experiments on Acoustic Absorption in Sand and Soil. J. Acoust. Soc. America, vol. 22, no. 4, July 1950, pp. 422-425.
- Tillotson, J. G.: Attenuation of Sound Over Snow-Covered Fields.
 J. Acoust. Soc. America, vol. 39, no. 1, Jan. 1966, pp. 171-173.
- Chessell, C. I.: Propagation of Noise Along a Finite Impedance Boundary.
 J. Acoust. Soc. America, vol. 62, no. 4, Oct. 1977, pp. 825-834.
- 40. Donato, R. J.: Impedance Models for Grass-Covered Ground. J. Acoust. Soc. America, vol. 61, no. 6, June 1977, pp. 1449-1452.
- 41. Jacques, J.; and Thomas, P.: Influence de la Présence d'un Plan Sur la Mesure de Spectres Contenant des Raies. SNEMCA communication presented at Deuxieme Colloque d'Acoustique Aeronautique (Paris), May 4-5, 1971.
- 42. Hoch, R.; and Thomas, P.: The Influence of Reflections on Turbo-Jet Acoustic Pressure Spectra. SNEMCA communication presented at First Colloquium of Aeronautical Acoustics (Toulouse), Mar. 6-8, 1968.

- 43. Miles, Jeffrey H.: Method of Representation of Acoustic Spectra and Reflection Corrections Applied to Externally Blown Flap Noise. NASA TM X-3179, 1975.
- 44. Miles, J. H.: Analysis of Ground Reflection of Jet Noise Obtained With Various Microphone Arrays Over an Asphalt Surface. NASA TM X-71696, [1975].
- 45. Miles, Jeffrey H.; Stevens, Grady H.; and Leininger, Gary G.: Application of Cepstral Techniques to Ground-Reflection Effects in Measured Acoustic Spectra. J. Acoust. Soc. America, vol. 61, no. 1, Jan. 1977, pp. 35-38.
- 46. Thomas, P.: Acoustic Interference by Reflection Application to the Sound Pressure Spectrum of Jets. NASA TT F-14,185, 1972.
- 47. Howes, Walton L.: Ground Reflection of Jet Noise. NASA TR R-35, 1959. (Supersedes NACA TN 4260.)
- 48. Pao, S. Paul; and Maestrello, Lucio: Evidence of the Beam Pattern Concept of Subsonic Jet Noise Emission. NASA TN D-8104, 1976.
- 49. Ingard, Uno; and Maling, George C., Jr.: On the Effect of Atmospheric Turbulence on Sound Propagated Over Ground. J. Acoust. Soc. America, vol. 35, no. 7, July 1963, pp. 1056-1063.
- Zipfel, G. G., Jr.; and DeSanto, John A.: Scattering of a Scalar Wave From a Random Rough Surface: A Diagrammatic Approach. J. Math. Phys., vol. 13, no. 12, Dec. 1972, pp. 1903-1911.
- 51. Wenzel, Alan R.: Smoothed Boundary Conditions for Randomly Rough Surfaces. J. Math. Phys., vol. 15, no. 3, Mar. 1974, pp. 317-323.
- 52. Bass, F. G.; Freulicher, V. D.; and Fuks, I. M.: Propagation in Statistically Irregular Waveguides. IEEE Trans. Antennas & Propag., vol. AP-22, no. 2, Mar. 1974, pp. 278-295.
- 53. Wenzel, Alan R.: Saturation Effects As ociated With Sound Propagation in a Turbulent Medium. AIAA Paper 75-546, Mar. 1975.

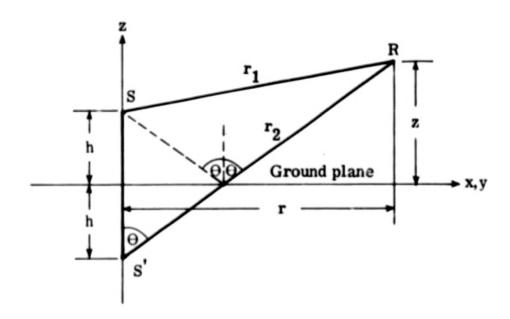


Figure 1.- Source and receiver geometry.

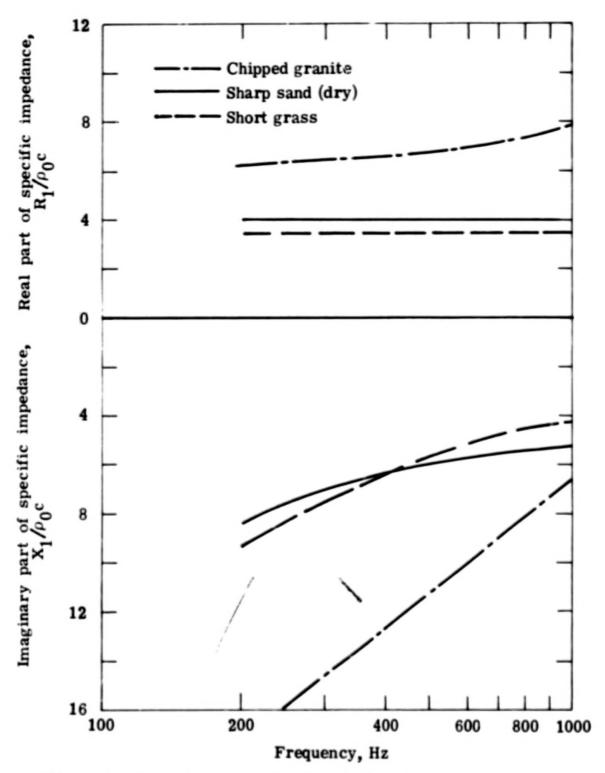


Figure 2.- Measured acoustic impedance of sand and grassland.

(From Dickinson and Doak, ref. 32.)

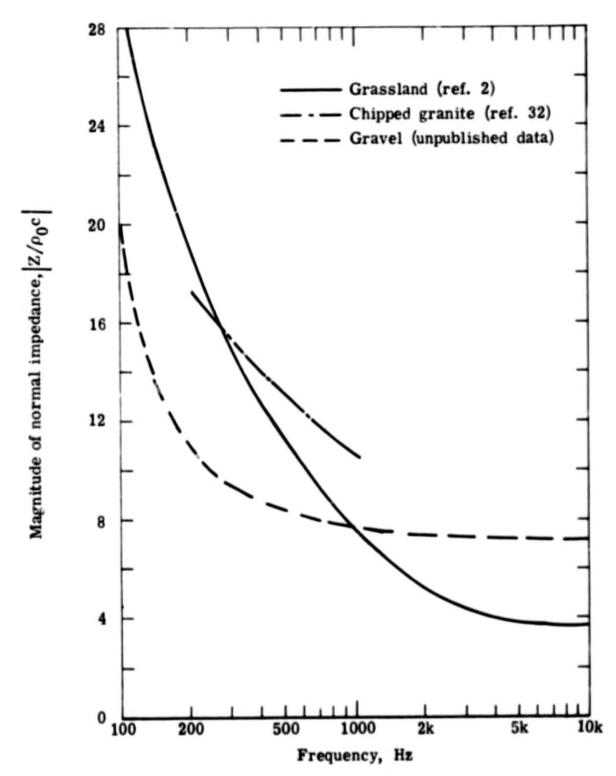


Figure 3.- Magnitude of acoustic impedance of grassland, chipped granite, and gravel.

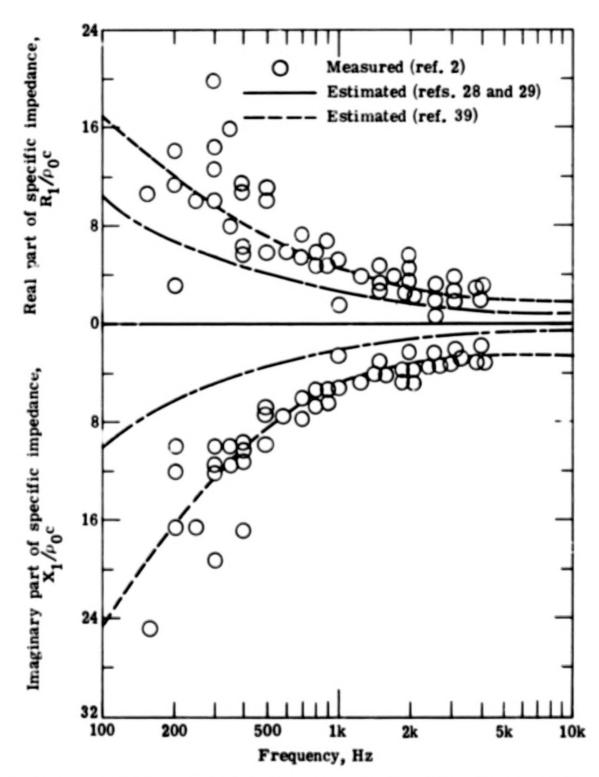


Figure 4.- Comparison of estimated acoustic impedance of ground by Delany and Bazley (refs. 28 and 29) and Chessell (ref. 39) with messured data reported by Embleton et al. (ref. 2).

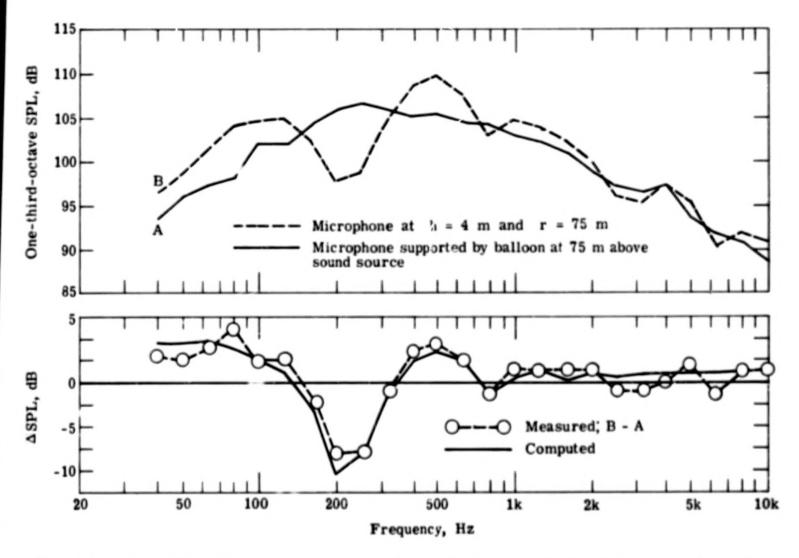


Figure 5.- Comparison of computed and measured ground effects on far-field noise measurement.

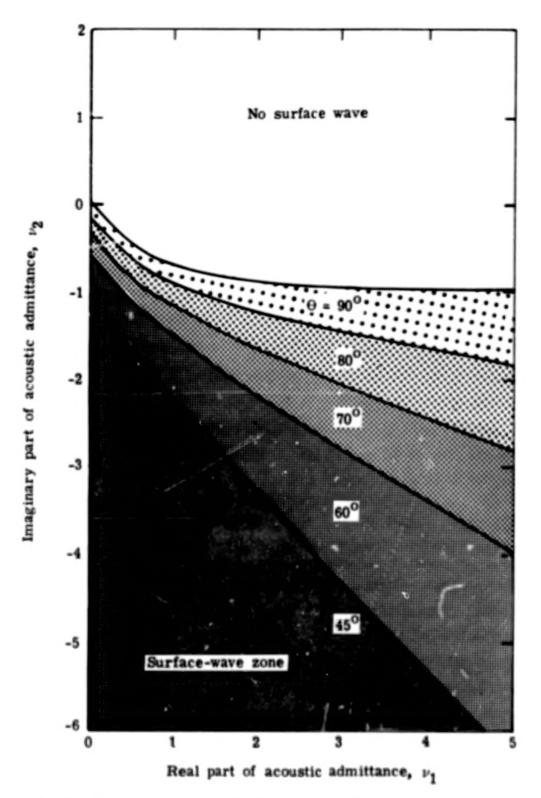


Figure 6.- Condition of existence of surface-wave component.

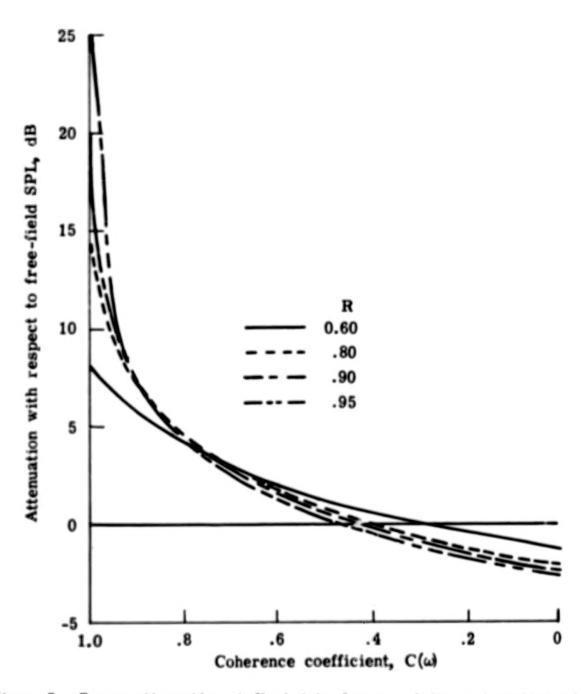


Figure 7.- Excess attenuation at first interference minimums at various values of reflection coefficient R and coherence coefficient $C(\omega)$.

1. Report No. NASA TP-1104	2. Government Access	sion No.	3. Recip	pient's Catalog No.		
4. Title and Subtitle			5. Repo	5. Report Date		
PREDICTION OF GROUND EFFECTS ON AIRCRAFT NOISE			Jan	nuary 1978		
		6. Perfo	6. Performing Organization Code			
7. Author(s)			8. Perfo	orming Organization Report No.		
S. Paul Pao, Alan R. Wenzel, and Paul B. Onc		Oncley	L-	-11833		
			10. Work	Unit No.		
9. Performing Organization Name and Address			50	05-03-21-01		
NASA Langley Research Center 1		11 Cont	ract or Grant No.			
Hampton, VA 23665						
				of Report and Period Covered		
12. Sponsoring Agency Name and Address			Te	Technical Paper		
National Aeronautics and Space Administration Washington, DC 20546		14 5000	soring Agency Code			
		14. 3501	saring Agency Code			
15. Supplementary Notes S. Paul Pao: Langley I Alan R. Wenzel: Instit Langley Research Cent Paul B. Oncley: MAN -	tute for Computer ter.					
A unified method is recommended for predicting ground effects on noise. This method may be used in flyover noise predictions and in correcting static test-stand data to free-field conditions. The recommendation is based on a review of recent progress in the theory of ground effects and of the experimental evidence which supports this theory. This review shows that a surface wave, a recently discovered effect, must be included sometimes in the prediction method. Prediction equations are collected conveniently in a single section of the paper. Methods of measuring ground impedance and the resulting ground-impedance data are also reviewed because the recommended method is based on a locally reactive impedance boundary model. Current practice of estimating ground effects are reviewed and consideration is given to practical problems in applying the recommended method. These problems include finite frequency-band filters, finite source dimension, wind and temperature gradients, and signal incoherence.						
17. Key Words (Suggested by Author(s))		10 Dissit -	ina Ctatament			
Ground effects			18. Distribution Statement Unclassified - Unlimited			
Sound propagation		THI COU				
Sound propagation						
				Subject Category 71		
19. Security Classif, (of this report)	20. Security Classif. (of t s	page)	21. No. of Pages	22. Price*		
Unclassified	Unclassified		40	\$4.50		